Application of Compressive Sensing to Gravitational Microlensing Data and Implications for Miniaturized Space Observatories

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Abstract. Compressive Sensing is a technique for simultaneous acquisition and compression of data that is sparse or can be made sparse in some domain. It is currently under intense development and has been profitably employed for industrial and medical applications. We here describe the use of this technique for the processing of astronomical data. We outline the procedure as applied to exoplanet gravitational microlensing and analyze measurement results and uncertainty values. We describe implications for on-spacecraft data processing for space observatories. Our findings suggest that application of these techniques may yield significant, enabling benefits especially for power and volume-limited space applications such as miniaturized or micro-constellation satellites.

1. Introduction

Compressive Sensing (CS) is a mathematical theory that enables acquisition of data samples at a much lower rate than the Nyquist rate. In this research, we describe the use of this technique for gravitational microlensing data. Many space flight instruments have limited data bandwidth and power consumption requirements. Modern science instrumentation rely increasingly on denser detector arrays and higher cadence observations resulting in tension between these demands. CS offers a possible solution by exploiting sparsity in the data to reduce data volume during acquisition. Note that CS is distinct from data compression schemes that have been successfully utilized for these applications previously and could be implemented as an additional measure to reduce science bandwidth demands.

We here make an initial assessment of the impact of application of CS to gravitational microlensing. Measurement using this astronomical technique, briefly described below, depends on observation of dense fields of distant stars due to the statistical rarity of any one microlensing event. Because the emission from most stars is relatively constant in the frequency range of interest, images of these dense fields are spatially sparse - most of the data in an image of millions of stars is of little scientific interest to the observer.

It is important to understand the phenomenology of gravitational microlensing so that we can assess the impact of application of CS on the extraction of germane information from the observed light curves. These curves can be quite subtle and beautiful and contain a great deal of information - information that is different for each observed

event. In this research, we study the effect of CS techniques on single lens systems as an initial case. In addition, we find optimal CS measurement parameters for a sample microlensing curve.

1.1. Gravitational Microlensing

Gravitational Microlensing refers to a relatively new observational technique to detect astronomical objects that are either faint or emit no light at all. Light from a distant source star departs the star radially and, in the absence of other masses in the Universe, would continue so without perturbation. However, if there is a concentrated mass between the source star and the observer, the rays are bent towards the mass resulting in a lens-like behavior. Flux from the source is preserved by this interaction so the magnification of the source star due to the mass results in a change in the apparent brightness of the source due to its proper motion. Such a magnification requires exquisite alignment of the source star and lensing body - thus the rarity of these events. The lensing body (hereafter referred to as the lens) may consist of one or more distinct masses. As the number of masses increase, the complexity of the light curve rapidly increases. Degeneracies in modeled lens mass distribution and source star multiplicity, size, dust extinction and maculation can cause misinterpretation of data - especially if the data are temporally sparse or miss key, high Fourier frequency characteristics.

In this work, we focus on microlensing curves generated by the most simple single mass lens systems. In a single lens system, magnification is mathematically normalized in units of 'Einstein ring' radius - a characteristic distance from the lensing body in the deflector plane that depends on lens and source distance and lens mass. The magnification of the source star at some time, t, is dependent on u_0, t_0, and t_e, where u_0 is the lens-source separation parameter, t_0 is the peak magnification time and t_e is the Einstein's ring radius crossing time [Seager (2010)].

1.2. Compressive Sensing

Compressive sensing is a mathematical theory for sampling at a rate much lower than the Nyquist rate, and yet, reconstructing the signal back with little or no loss of information. Compressive sensing works well for sparse signals. Hence, if a signal is not sparse in the sampling domain, it can be transformed to a sparse domain, where reconstruction can take place. We assume x to be k-sparse signal of length N, ϕ represents the measurement matrix, and ψ is the transform domain to obtain sparsity. Both ϕ and ψ are of size M x N. The measurements,y, obtained by the projection of $\phi\psi$ onto x is an M length vector, where M << N. Parameters ϕ and ψ are known, and y is the acquired measurements vector. The original signal, x, the equation, $y = \phi\Psi x = \Phi x$, can be reconstructed by solving an optimization problem by minimizing the L1 norm [Eldar & Kutynoik (2012)]. In gravitational microlensing applications, the spatial domain is assumed to be sparse. Hence, Ψ can be represented by an Identity matrix.

The reconstruction algorithm solves for x, given y and Φ. Various reconstruction algorithms exists to solve such optimization problems. In this research, a new algorithm, conic optimization via operator splitting and homogeneous self-dual embedding, is used. A software python package is used in the simulations [O'Donoghue et al. (2016)].

2. Simulations

2.1. Experiment Setup

All simulations are performed in python. The source star has a Gaussian point spread function (PSF) with mean 0 and standard deviation of 0.2. Lens-source separation parameter, u_0, is set to 0.1. Time parameters, t_0 and t_e, are set to 15 and 30, respectively. Microlensing curves are generated over 30 times samples, from t=1 to t = 30. Measurement matrix, ϕ, consists of Bernoulli random variables of values 0 and 1. At each time, t, the number of measurements, M, are varied from 2% of N to 6% of N. All simulations for each t and M value are run over 100 Monte Carlo simulations where the Bernoulli random measurement matrix is varied. A 3-pixel radius is used to calculate the total flux of the source star in the image at any given time.With the given parameters, a spatial domain image and a time domain signal representing the microlensing curve are shown in Figure 1.

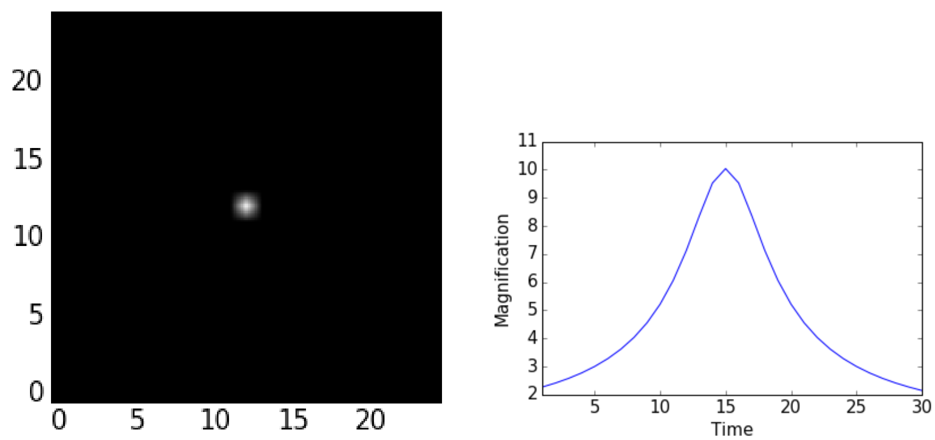

Figure 1. *Left:* Spatial Domain Image at t_0 (t= 15). *Right:* Total Flux over the center pixel with a 3-pixel radius at each time, t

2.2. Results

Simulation results show that a microlensing curve can be accurately reconstructed using compressive sensing techniques by capturing changes in magnification over all time samples. The minimum difference in magnification is between t=29 and t=30 with a value of 0.12 units of flux and the change in magnification at t_0 is 0.5 units of flux. To ensure that the microlensing curve is accurately captured, the reconstructed signal resolution at time t=29 must be within 0.12 units of the original magnification curve and at t=14 within 0.5 units. In table 1, the difference in magnitude between the reconstructed signal and original signal at t=14 and t=29 is shown. The error difference must be less than the change in magnitude of the original signal at those times. Figure 2 (Left) shows the error bars of the flux over 3-pixel radius at each time for 100 Monte Carlos simulations. Figure 2 and Table 1 show that 4% of N is the smallest value of M required to correctly reconstruct the shape of the microlensing curve while having low uncertainty values.

t = 14	t = 29	Avg. sdev over all t
4.07	0.86	1.60
0.52	0.044	0.52
4.15×10^{-5}	5.77×10^{-5}	9.64×10^{-4}
1.81×10^{-4}	2.58×10^{-4}	7.76×10^{-4}
4.19×10^{-5}	5.36×10^{-6}	7.32×10^{-4}

Table 1. Reconstruction Error (magnitude difference) and average standard deviation for 2% to 6% measurements, accordingly (top to bottom)

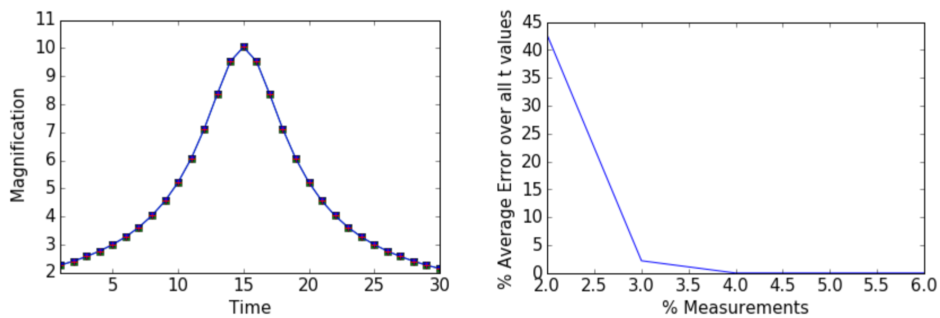

Figure 2. *Left:* Original signal and reconstructed signal for Total Flux over a 3-pixel radius at each time, t. Squares = Original, Triangles = Reconstructed, + = standard deviation over 100 simulations. Reconstructed signal overlaps original signal with very low uncertainty. *Right:* % Average Error over all t for center pixel with a 3-pixel radius for varying %Measurements ($\frac{M}{N}$ x 100) from 2% of N to 6% of N

3. Conclusion and Future Work

This work suggests that CS has the potential to reduce data volume while retaining important information about the observed object. Space observatories with data bandwidth, volume and power limitations may benefit from this new technique. The simulations in this work show that we can reconstruct a clean image of a source star with only one pixel having significant value using only 4% of the number of samples we would otherwise need using traditional methods. Future work will consists of analyzing microlensing events for more complex lenses and for non-differenced images. Point spread function and convolution kernels for differencing imaging will also be studied as applicable to CS techniques.

References

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